Evidence for self-organized criticality in the Bean critical state in superconductors

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The time dependence of the magnetization of a type-II superconductor in the Bean critical state is studied. It is found that evolution occurs in the form of bursts, consistent with a model exhibiting self-organized criticality. The distribution of step sizes follows a power law with an exponent of $\alpha \simeq 2$. At high temperatures the distribution is Gaussian-like, as would be expected in an equilibrium situation. This may allow the experimental study of the occurrence of criticality. [S1063-651X(98)03508-9]

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In a type-II superconductor an applied magnetic field may penetrate into the bulk of the superconducting sample. This happens in the form of quantized lines of magnetic flux. In equilibrium, these flux lines (or vortices) form a regular hexagonal lattice. Defects in the crystal structure, suppressing the superconductivity, may be favoring the position of a vortex line, effectively pinning it. Thus when the superconductor is driven out of the equilibrium situation it is these pinning sites that determine its behavior [1]. This can be achieved, for instance, by applying the field in the superconducting state, such that penetrating vortices get pinned near the surface. These pinned vortices then repel other entering vortices, leading to an extremal dynamics of the whole system, where the most weakly pinned flux lines depin in order to give way to the pressure exerted by the penetrating flux lines. As is the case in a sandpile, this will eventually lead to a situation with a constant gradient (here of magnetic field) over the boundary. Such a situation has already been described by Bean some 30 years ago [2]. Recently it has been noted that this state may be an example of self-organized criticality (SOC) [3] and thus be governed by power-law distributions in its dynamics [4,5].

SOC has been invoked to explain a host of natural phenomena, ranging from earthquakes [6] to biological macroevolution [7–9], that take place far from thermodynamic equilibrium. In the case of earthquakes this is borne out by the Gutenberg-Richter law, describing a power-law distribution in the strength of earthquakes. This is taken as a sign of a critical state, where there are fluctuations on all scales as given by a power-law distribution. In macroevolution, the discovery of punctuated equilibrium in the fossil record by Gould and Eldredge [10] has led to speculations that similar to the Earth's crust, the biosphere could be in a critical state leading to the observed power-law behavior in the extinction pattern. In the original proposition of SOC the picture of a sandpile was invoked to convey a physical picture of the computer model [3]. Consider a table onto which we drop sand grains at random. After a certain growth period, there will be a sandpile of definite slope that no longer changes with time. In that case the sandpile is in a critical state, where the slope is kept constant by occasional bursts of sandslides (avalanches), whose size distribution takes the form of a power law. As there is no fine-tuning of any parameters involved to obtain the critical state this is called a self-organized critical state. In this discussion we have not yet considered how this SOC state is reached. More information on this problem has been gained recently by studying the extremal dynamics governing depinning processes or the evolution by natural selection [8,11,12,9]. In the language of evolution, the most unfit species is most probably extinct. Its ecological niche is subsequently occupied by another species, thus changing the interaction between species. This may make other species unfit leading to a chain reaction of extinctions [13]. With time, all species will increase in fitness, such that in the end their fitness is above a certain threshold for all of them. When this is the case, the SOC state is reached, where small random fluctuations may induce system-spanning extinction events that obey a power-law size distribution [11]. In macroevolution, this process will depend strongly on the rate of mutation of species determining whether the interspecies interactions change fast enough to have a system-spanning effect [8]. A high rate of mutation would effectively decouple the ecosystem into independent subsystems, thus prohibiting mass extinctions. A similar consideration holds in the depinning of vortices in the Bean state. The most weakly pinned vortices are the ones most likely depinned. However, the change of such a vortex results in a different pinning landscape for all the others, thus changing the pinning energy of all vortices with time, until again a SOC state is reached, in which all vortices are pinned stronger than a certain threshold. As in the case of macroevolution, there is a mechanism working against this scenario, which in this case is that of thermal fluctuations. Considering in the extreme case, if the temperature connected with the thermal fluctuations of the vortices exceeds the threshold pinning barrier, vortices above the threshold would be unpinned, effectively decoupling the system. Thus investigating the behavior of the magnetization in the Bean state at different temperatures may give insight into the occurrence of the SOC state. We will discuss this further below.

The measurements were carried out using a commercial superconducting quantum interference device magnetometer by Quantum Design (San Diego). The sample used consisted of a single crystal of $Bi_2Sr_2CaCu_2O_8$ (BSCCO), grown using a floating zone method described elsewhere [14]. We used a BSCCO crystal for our sample as the behavior of vortices in this material is by now well known due to very intense theoretical and experimental investigations over the past decade [16]. The main feature of the vortex behavior is that BSCCO exhibits predominant point pinning at low temperatures, but also shows many other well known pinning behaviors depending on the applied field and the temperature [1,15]. At high temperatures, the vortex lattice is even thought to melt, giving rise to an equilibrium behavior of the vortices [16]. Thus studying BSCCO gives good control on the pinning strengths and mechanisms. The experiment is carried out by first cooling the sample in zero field to low temperatures before the desired field is applied (ZFC). This way there is a constant supply of energy from the applied magnetic field effectively keeping up the nonequilibrium situation, resulting in the buildup of a critical state in which vortices enter the superconductor in bursts (or avalanches). This, however, would not be the case were the experiment carried out in the opposite direction. In that case the sample is cooled in a field that is subsequently taken to zero at low temperature. Thus there would be no constant energy supply for the sample to stay in a nonequilibrium state and it would exhibit a relaxation to the equilibrium that is not governed by avalanche behavior. We will return to this in the discussion of the results.

The entrance of flux lines is measured by the change in the magnetic moment of the superconducting sample due to the effective reduction of the field expulsion. This means that we can assess the size of a ''vortex avalanche'' from the magnitude of the change in magnetization of the sample. This behavior should present a model example of ''extremal dynamics'' [17], as the intervortex interactions as well as those between vortices and pinning sites are such that the magnetic field gradient exerts a force on the pins flux lines resulting in a depinning of the most weakly pinned vortex. Subsequent to such a depinning event the intervortex interactions may destabilize other flux lines, leading to big changes in the magnetization of the sample. In the language of sandpiles, this would correspond to grains of sand falling off the table. On the other hand, in the language of macroevolution, such a change in magnetization would correspond to an extinction event, where the magnitude of the magnetization step corresponds to the size of the extinction (number of species that have gone extinct in the event). The size distribution in both cases follows a power law given by

$$p(S) \sim S^{-\alpha}, \tag{1}$$

where the exponent α is characteristic of the problem and ranges from 1 to 2.5. In the case of macroevolution in particular, $\alpha \simeq 2$ [18].

In order to measure the avalanche behavior we performed careful measurements of the time evolution of the magnetization of a BSCCO sample over very long time periods, using a scanning length of 1 cm to minimize magnetic field inhomogeneities over the sample. By varying the temperature, we may determine the transient period it takes until the critical state is reached. At very high temperatures this is never the case, as the flux lines then form a liquid that is always in thermal equilibrium and hence does not show the avalanche behavior emblematic of self-organized criticality. However, at intermediate temperatures a critical state is still reached after a transient period, depending on the temperature. This behavior can again be formulated in the language of sandpiles, where the temperature would correspond to the wetness of the sand in the pile resulting in increased inertia

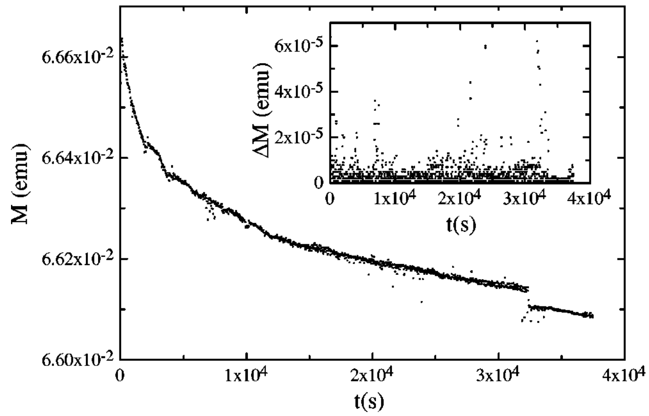

FIG. 1. Time evolution of the magnetization measured at a temperature of 5 K. Note the occasional steps in the magnetization, associated with avalanches of magnetic flux lines penetrating the sample. The size distribution of these avalanches obeys a power law as shown in Fig. 2.

of the sand grains for increased temperature. On the other hand in the language of macroevolution, increased temperature would correspond to an increased rate of mutation, as discussed above. Such a point of view has already been taken by Sneppen in a self-organized critical model of macroevolution using extremal dynamics [8].

In Fig. 1 the time evolution of the magnetization is given at a temperature of 5 K. As can be seen, there appear big changes over short times that give a steplike behavior. Due to the poor time resolution of the apparatus ($\approx$20 s per measurement), only big steps can be observed. This arises from the fact that the mean change in magnetization over a measuring period gives a lower limit to the observed step size. We furthermore determine the size distribution of magnetization changes by calculating the difference between two subsequent time steps (shown in the inset of Fig. 1), which we take to be magnetization steps. Then calculating the probability distribution of these changes in magnetization gives the result shown in Fig. 2. For changes bigger than a certain threshold given by the time resolution of the magnetometer as discussed above, this size distribution turns out to be a power law

$$p(\Delta M) \sim \Delta M^{-\alpha}, \tag{2}$$

with an exponent of $\alpha \simeq 2$. This is similar to the exponent found by slowly ramping the field [5]. The exponent is also exactly the same as that observed in the fossil record for the size distribution of extinction events in biological evolution. This can be taken as a sign of the appearance of a SOC state in the superconductor leading to the observed power-law behavior in the time evolution of the magnetization as described above. To ascertain that such power-law behavior does not stem from the special measuring process, such as small inhomogeneities in the applied field or other spurious effects, we can perform the same measurement in the vortex liquid state. At these high temperatures, vortices will always be in thermal equilibrium prohibiting the occurrence of a critical state. Thus, in that situation there should be no power-law behavior in the size distribution of the magnetization changes. As can be seen in Fig. 3, the size-distribution

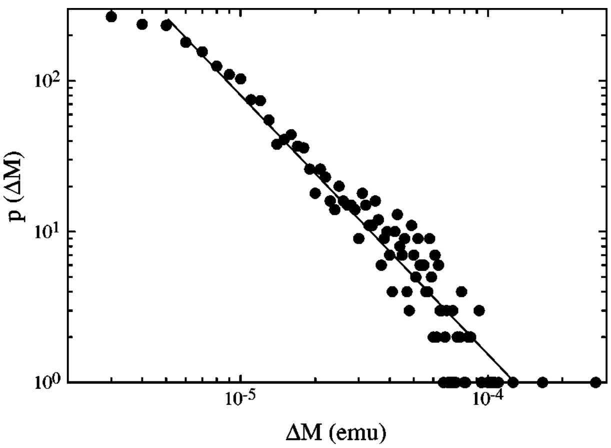

FIG. 2. Size distribution of the differences in magnetization at 5 K (shown in Fig. 1). The distribution follows a power law with an exponent of $\alpha \simeq 2$. This is a strong indication of the appearance of a SOC state in the superconductor (see the text). The observed exponent is identical to that found in the size distribution of extinction events in biological macroevolution [10,18] from the fossil record.

in that case is close to a Gaussian distribution, as should be expected for such a situation. In the same figure, we also include data from a measurement cooled in a field. As discussed above, that case as well should not exhibit the power-law behavior of a SOC state, as there is no constant energy supply keeping the superconductor out of equilibrium. This should rather reflect a relaxation to the equilibrium state, where the size distribution of the magnetization changes is again determined by random noise and should therefore be close to a Gaussian distribution.

Let us now turn to the effect of temperature on the occurrence of the SOC state. As discussed above, increasing the temperature corresponds to an effective decoupling of the system, increasing the transient period the system takes to reach the SOC state [11]. In Fig. 4(a) we show the time dependence of the magnetization at a temperature of 20 K. It can be seen from the figure that at short times, the evolution is continuous, with a rather big overall change in the magnetization. After a long (≈ 24 h) transient period, however, shown in Fig. 4(b), the evolution now place in bursts, as was the case at 5 K. Here again, the minimum changes are determined by the time resolution resulting in an effective averaging of very small changes. Such a behavior is expected from an analysis of the evolution model of Bak and Sneppen

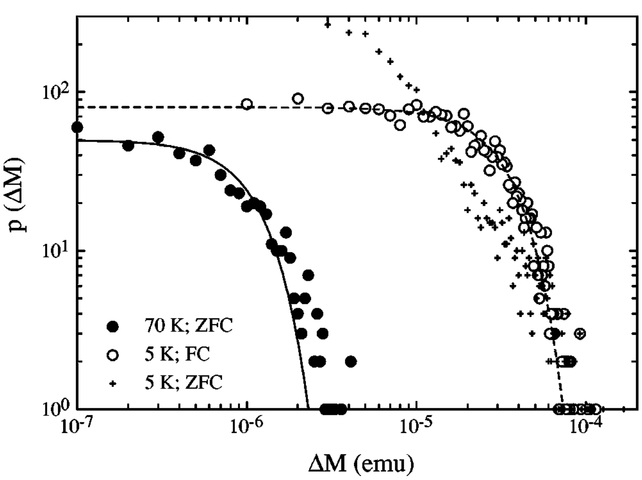

FIG. 3. Size distribution of magnetization changes at high temperatures (70 K, in the vortex liquid state) (full circles) and cooled in a field (FC) at low temperature. In the FC case, there is no constant energy supply leading to a relaxation to an equilibrium state. Both cases are thus governed by equilibrium dynamics where the size distribution is expected to be of Gaussian form, as indicated by the lines through the data. These findings also show that the results in Fig. 2, shown again as crosses for comparison, are intrinsic to the sample and do not arise from the apparatus (see the text).

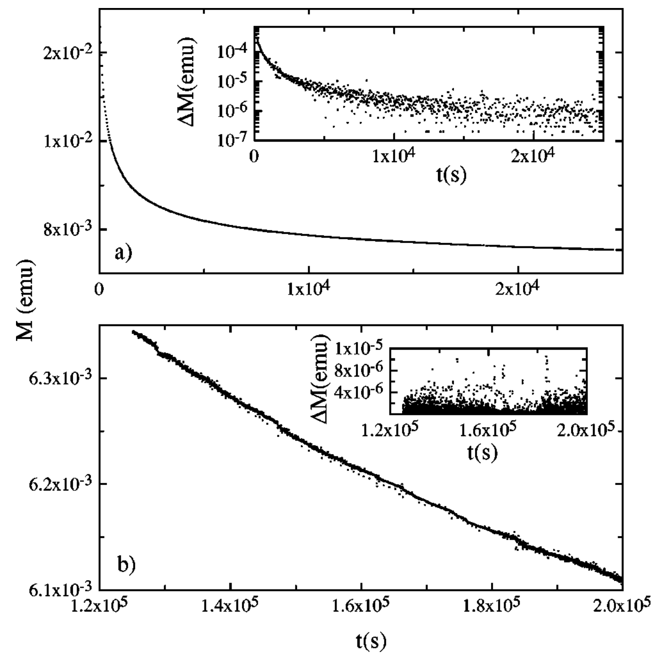

FIG. 4. Time evolution of the magnetization measured at a temperature of 20 K. At short times (a), during the transient period before the SOC state is reached, the time dependence is smooth. Furthermore, the average activity as given by the differences in magnetization goes to zero during the transient (see the text). At long times (b), the SOC state has been reached, resulting in steplike behavior in the time evolution. The corresponding size distributions are given in Fig. 5.

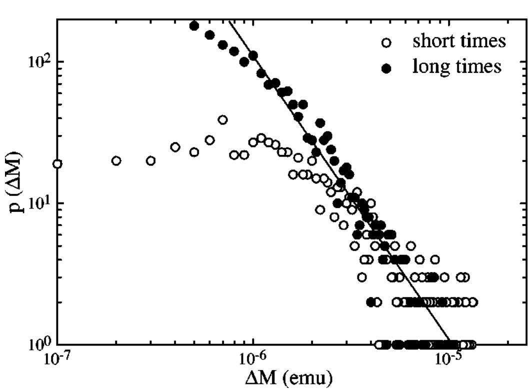

FIG. 5. Size distribution of the magnetization changes for short and long times at 20 K. At short times (open circles), during the transient period, the distribution is Gaussian-like, as in the equilibrium cases. At long times (full circles), however, where the time evolution is similar to that at low temperatures, the size distribution follows a power law, with the same exponent as at 5 K.

[7], predicting smooth behavior in the beginning getting more and more intermittent as the SOC state is reached [11]. Another proposition drawn from the analysis of the Bak-Sneppen model is that after the transient period where there is overall activity involved in the buildup of the critical state the average number of active sites should be constant. This evolution to the critical state can be observed in the inset of Fig. 4(a), where the time dependence of the difference in magnetization is given at a temperature of 20 K. As can be seen in the figure, the average activity approaches zero at long times. This is in good accord with the prediction of stationarity by Paczuski *et al.* [12], given by

$$\langle n(s)\rangle \sim s^{\eta}, \tag{3}$$

with $\eta=0$.

Another difference in the behavior at short and long times associated with reaching the SOC state can be observed in Fig. 5. There we show the size distribution of events for both cases. It can be seen in the figure that with increasing time the size distribution changes from being Gaussian-like to approximating a power law, with the same exponent as at low temperatures. This again can be taken as a measure of how the critical state is reached. Thus a systematic study with temperature might give a critical value of temperature, above which the SOC state cannot be reached [8].

In conclusion, we have studied in detail the long time behavior of the magnetization of a superconducting sample in the Bean state. We have found that the evolution of the magnetization with time takes place in bursts that follow a power law in the size distribution with an exponent of $\alpha \simeq 2$. This is argued to arise from the buildup of a SOC state in the superconductor, due to the extremal dynamics in the behavior of the vortices. This dynamics is similar to that of evolution in ecosystems, which is supported by the fact that the exponent observed in the size distribution of extinction events from the fossil record is the same as that observed here in the Bean state. Furthermore, the effect of temperature on the Bean state is analogous to that of the rate of mutation in evolution. Thus studying the Bean state gives experimental control and insights on how the SOC state is reached.

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